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Sound Generation by a Ring Vortex–Shock Wave Interaction

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Introduction

FLOWS with supercritical speeds, which are characterized by shock waves, are often accompanied by vortices of various intensities. Both structures may interact, leading to impulsive noise radiation. This phenomenon is observed in numerous external and internal transonic flows. In the case of helicopter or cascade blades operating at high-subsonic-flow Mach numbers, a shock that appears on the low-pressure blade surface may interact with the vortex street from preceding blades. Similarly, in an underexpanded jet, the vortices at the shear layer periodically pass through the shock-wave pattern of the jet, causing intensive noise.[1]

These aerodynamic problems have motivated many investigators to elucidate the physics of the shock–vortex interaction process. Theoretical and experimental investigations have been done. In the theoretical studies, analytical[2,3] and numerical methods[4–6] were considered, and have been used to predict both the sound wave formed at the region of interaction and the shock-wave deformation when it passes through the vortex.

The theoretical investigations of the interaction process were performed for a cylindrical vortex and a plane-incident shock wave. This case also was examined experimentally by Dosanjh and Weeks[7] and Naumann and Hermans,[8] who used a starting vortex shed from the trailing edge of the airfoil when the shock wave passes over it. The main result of both the theoretical and the experimental studies is the finding that the two-dimensional vortex–shock wave interaction produces a quasicylindrical sound wave (Fig. 1) of nonuniform strength; the strength of the sound wave decreases along its front with increasing distance from the triple point A. However, to some extent a different wave pattern can be expected when the ring vortex–shock wave interaction appears in an axisymmetric flow. Also, when the flow in a jet occurs along the axes of the ring vortex, it can influence the sound-wave configuration. This problem is studied in the present paper for the head vortex of a starting jet.

Apparatus

A conventional shock tube, internal diameter $D = 50$ mm, was used in the experiments. A starting jet was incident on a perpendicular wall placed at a distance of two tube diameters from the shock-tube exit. The free jet was visualized by means of a schlieren system in which a spark source of flash duration about 1 μs was used for a illumination. The shock-wave Mach number in the shock tube was determined by measuring the time traveled by the shock wave over a test distance of 100 mm.

Results

A set of photographs (Fig. 2) shows the successive phases of the vortex–shock wave interaction process for the jet-flow Mach number $M = 0.65$ (the jet-flow Mach number was calculated on the basis of the shock-wave Mach number). The first photograph shows a bow shock wave reflected at the wall and a head vortex just before interaction. The central part of the shock wave moves slowly because of the opposite flow caused by the jet (photograph 2). Because of this effect, the shock wave loses its previous shape, which is analogous to the two-dimensional flow case.[8] However, in contrast to the two-dimensional case, the shock deformation is now much stronger owing to the high velocity of the jet flow. In the stage shown in photograph 4, three shock-wave elements can be distinguished: 1) a nearly normal shock wave in the vortex plane moving slowly upstream; 2) an undisturbed bow shock wave outside

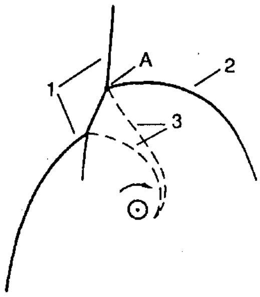

Fig. 1 Wave pattern for a cylindrical vortex–plane shock wave interaction (from Ref. 8): 1) shock front, 2) sound wave, 3) contact surface, and A) triple point.

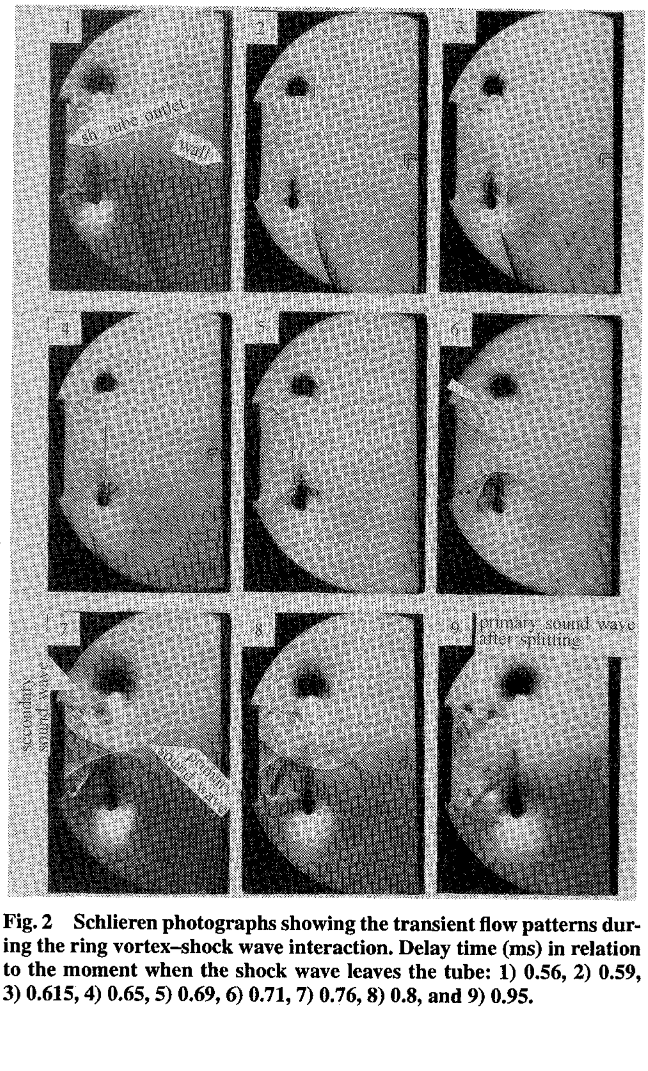

Fig. 2 Schlieren photographs showing the transient flow patterns during the ring vortex–shock wave interaction. Delay time (ms) in relation to the moment when the shock wave leaves the tube: 1) 0.56, 2) 0.59, 3) 0.615, 4) 0.65, 5) 0.69, 6) 0.71, 7) 0.76, 8) 0.8, and 9) 0.95.

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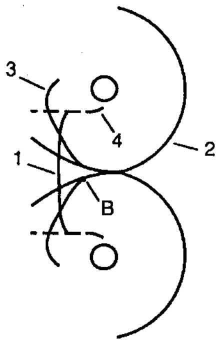

Fig. 3 Schematic diagram of a transient flow pattern for a ring vortex–shock wave interaction: 1) shock wave, 2) primary sound wave, 3) secondary sound wave, 4) shear layer, and B) triple point.

the vortex; and 3) a toroidal wave (a sound wave) around the vortex core.

The normal shock wave is crossed by the toroidal wave (photograph 5). Since the radius of the vortex ring continuously increases, it remains larger than the radius of the jet. As a consequence, a part of the normal shock wave between the vortex core and the shear layer can move faster upstream owing to lower flow velocity in that region (arrow in photograph 6). This leads to shock-wave diffraction and eventually to the secondary sound-wave formation visible in photographs 7 and 8. Photograph 7 shows the stage when the toroidal wave has just reached the axes of the system. Now, the normal shock wave is accompanied by two sound waves: the primary (toroidal) wave and the secondary one. Both waves join at the triple point B (Fig. 3). The sound wave, after reflection at the axis of symmetry, spreads out in the surrounding space as a secondary sound pressure pulse (the primary one corresponds to the primary sound wave). It splits into two parts while it is passing through the vortex (photograph 9), analogous to the shock wave in the stage shown in photograph 4.

The photographs described above provide additional information on the flow under consideration. They show small vortices at the shear layer being sucked by a head vortex and in this way strengthening the head vortex. The sound-wave motion in the radial direction induces the shear layer instabilities, which are manifested in formation of discrete vortices (see photograph 9). However, the details of this process are difficult to explain on the basis of the photographs under discussion.

Conclusion

The vortex–shock wave interaction process in an axisymmetric flow leads to a complicated sound-wave pattern. It consists of the primary toroidal wave that appears as the shock wave passes through the ring vortex and a secondary wave that arises, with some delay, because of the strongly nonuniform flow velocity in the vortex plane. The primary wave diffracts while it crosses the ring vortex.

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Boundary-Layer Transition Due to Isolated Three-Dimensional Roughness on Airfoil Leading Edge

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Introduction

ICE roughness on an airfoil initially forms very near the leading edge in a region of favorable pressure gradient and rapidly grows to exceed the height of the boundary layer. Understanding the effect that hemispherical ice roughness has on the boundary-layer flow and heat transfer is a very important part of ice accretion physics. This investigation sought to answer two questions: 1) What is the critical-roughness Reynolds number for elements protruding out of the boundary layer and experiencing a pressure gradient and 2) how large is the transitional region behind a critical element?

Most classic experiments on roughness involved small roughness contained within a laminar boundary layer on a flat plate (no pressure gradient).[1–3] The roughness Reynolds number was defined as $Re_k = U_k k/v$, where k is the height of the roughness, v the kinematic viscosity, and U_k the velocity of the undisturbed boundary-layer flow at the height of the roughness. The critical- roughness Reynolds number, $Re_{k,\text{crit}}$, was normally defined as the Re_k where transition occurred immediately behind the roughness element. Although a wide range of values for $Re_{k,\text{crit}}$ has been reported, a value of 600 generally has been accepted and used in practice for all roughness, particularly for fixing transition on wind tunnel models.[4] In reality, transition does not occur at the element, but approaches the element asymptotically as Re_k is increased past the critical value.[5]

Braslow et al.[4] and von Doenhoff and Horton[6] noted an increase in $Re_{k,\text{crit}}$ for distributed roughness near the leading edge of an airfoil. $Re_{k,\text{crit}}$ values increased to approximately 1200 very near the leading edge, in part because of the pressure gradient and/or k/δ effects (δ is the undisturbed boundary-layer thickness). However, Smith and Clutter[2] as well as Peterson and Horton[7] found little or no effect of pressure gradient alone. Peterson and Horton also reported no effect from varying k/δ for roughness within the boundary layer. However, Morkovin[5] showed that for a fixed cylinder protruding out of the boundary layer, the flowfield instabilities that appear to lead to transition are different from those for the cylinder within the boundary layer. Presumably, this would alter the value of $Re_{k,\text{crit}}$, or possibly necessitate a new definition of $Re_{k,\text{crit}}$ for roughness protruding out of the boundary layer.

Thus, the motivation for the current study was to increase our understanding of $Re_{k,\text{crit}}$ and the transition region caused by large leading-edge roughness. The current investigation clearly revealed the inadequacy of assuming $Re_{k,\text{crit}} = 600$ near the airfoil leading edge. $Re_{k,\text{crit}}$ values in excess of 2000 were found, and the experiment suggests that even larger values may exist closer to the leading edge. The transition region behind the roughness extended well downstream before developing into a fully turbulent boundary layer and, in some instances, the transition region actually grew in length as Re_k (and, as a result, k/δ) was increased. This Note briefly describes these results. A more complete discussion of this research can be found in Cummings[8] and Bragg et al.[9]

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